Radio Frequency Circuit Design. W. Alan Davis, Krishna Agarwal
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CHAPTER SIX

Transmission Line Transformers

6.1 INTRODUCTION

The subject matter of Chapter 3 was impedance transformation. This subject is taken up here again, but now with more careful attention given to the special problems and solutions required for RF frequency designs. The discrete element designs described previously can be used in RF designs with the understanding that element values will change as frequency changes. The alternative to discrete element circuits are transmission line circuits. The classical microwave quarter wavelength transformer can be used up to hundreds of GHz in the appropriate transmission line medium. However, at 1 GHz, a three-section quarter wavelength transformer would be a little less than a meter long! The solution lies in finding a transformation structure that may not work at 100 GHz but will be practical at 1 GHz.

The conventional transformer consists of two windings on a high-permeability iron core. The flux, ϕ, is induced onto the core by the primary winding. By Faraday's law the secondary voltage is proportional to $d\phi/dt$. For low-loss materials, the primary and secondary voltages will be in phase. Ideal Transformers have perfect coupling and no losses. The primary-to-secondary voltage ratio is equal to the turns ratio, n, between the primary and secondary windings, namely $V_p/V_s = n$. The ratio of the primary to secondary current is $I_p/I_s = 1/n$. This implies conservation of power, $V_pI_p = V_sI_s$. As a consequence the impedance seen by the generator or primary side in terms of the load impedance is

$$Z_G = n^2 Z_L$$

When the secondary side of the ideal transformer is an open circuit, the input impedance of the transformer on the primary side is ∞.

In a physical transformer the ratio of the leakage inductances on primary and secondary sides is $L_p/L_s = n$. For the ideal transformer, L_p and L_s approach

∞, but their ratio remains finite at $L_p/L_s = n$. The physical transformer has an associated mutual inductance, $M = k\sqrt{L_p L_s}$, where k is the coupling coefficient. The leakage inductance together with the interwire capacitances limits the high-frequency response. The transmission line transformer avoids these frequency limitations.

6.2 IDEAL TRANSMISSION LINE TRANSFORMERS

It was found earlier, in Chapter 2, that inductive coils always come with stray capacitance. It was this capacitance that restricted the frequency range for a standard coupled coil transformer. The transmission line transformer can be thought of as simply tipping the coupled coil transformer on its side. The coil inductance and stray capacitance now form the components for an artificial transmission line whose characteristic impedance is

$$Z_0 = \sqrt{\frac{L}{C}} \tag{6.1}$$

The transmission line can be used, in principle, up to very high frequencies, and in effect it reduces the deleterious effects of the parasitic capacitance. The transmission line transformer can be made from a variety of forms of transmission lines such as a two parallel lines, a twisted pair of lines, a coaxial cable, or a pair of wires on a ferrite core. The transmission line transformer can be defined as having the following characteristics:

1. The transmission line transformer is made up of interconnected lines whose characteristic impedance is a function of such mechanical things as wire diameter, wire spacing, and insulation dielectric constant.
2. The transmission lines are designed to suppress even mode currents and allow only odd-mode currents to flow (Fig. 6.1).
3. The transmission lines carry their own "ground," so transmission lines relative to true ground are unintentional.
4. All transmission lines are of equal length and typically $< \lambda/8$.
5. The transmission lines are connected at their ends only.
6. Two different transmission lines are not coupled together by either capacitance or inductance.

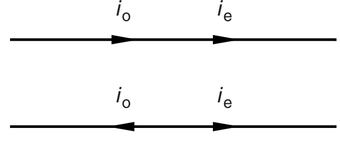

FIGURE 6.1 A two-wire transmission line showing the odd- and even-mode currents.

7. For a short transmission line, the voltage difference between the terminals at the input port is the same as the voltage difference at the output port.

Some explanation of these points is needed to clarify the characteristics of the transmission line transformer. In property 2, for a standard transmission line the current going to the right must be equal to the current going to the left in order to preserve current continuity (Fig. 6.1). Since only odd-mode currents are allowed, the external magnetic fields are negligible. The net current driving the magnetic field outside of the transmission line is low. The third point is implied by the second. The transmission line is isolated from other lines as well as the ground. The equality of the odd mode currents in the two lines of the transmission line as well as the equivalence of the voltages across each end of the transmission line is dependent on the transmission line being electrically short in length. The analysis of transmission line transformers will be based on the given assumptions above.

As an example consider the transmission line transformer consisting of one transmission line with two conductors connected as shown in Fig. 6.2. The transformation ratio will be found for this connection. Assume first that v_1 is the voltage across R_G and i_1 is the current leaving the generator resistance:

1. i_1 is the current through the upper conductor of the transmission line.
2. The odd-mode current i_1 flows in the opposite direction in the lower conductor of the transmission line.
3. The sum of the two transmission line currents at the output node is $2i_1$.
4. The voltage at the output node is assumed to be v_o. Consequently the voltage at left side of the lower conductor in the transmission line is v_o above ground.
5. On the left-hand side, the voltage difference between the two conductors is $v_1 - v_o$.

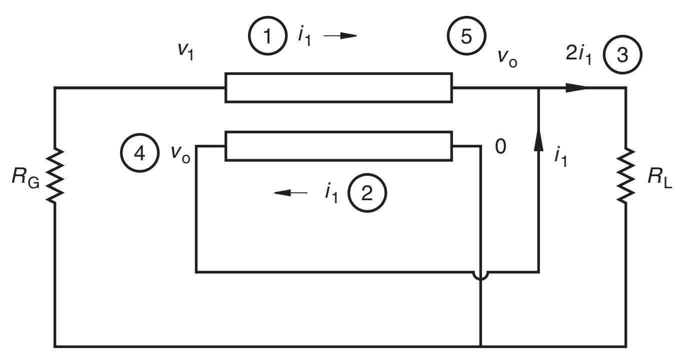

FIGURE 6.2 Analysis steps for a transmission line transformer.

This is the same voltage difference on the right hand side. Consequently,

$$v_o - 0 = v_1 - v_o$$
$$v_o = \frac{v_1}{2}$$

If $R_G = v_1/i_1$, then

$$R_L = \frac{v_o}{2i_1} = \frac{v_1/2}{2i_1} = \frac{R_G}{4} \tag{6.2}$$

This 4 : 1 circuit steps down the impedance level by a factor of 4.

A physical connection for this transformer is shown in Fig. 6.3 where the transmission line is represented as a pair of lines. In this diagram the nodes in the physical representation are matched to the corresponding nodes of the formal representation. The transmission line is bent around to make the $B-C$ distance a short length. The transmission line, shown here as a two-wire line, can take a variety of forms such as coupled line around a ferromagnetic core, flexible microstrip line, or coaxial line. If the transformer is rotated about a vertical axis at the center, the circuit shown in Fig. 6.4 results. Obviously this results in a 1 : 4 transformer where $R_L = 4R_G$. Similar analysis to that given above verifies this result. In addition multiple two-wire transmission line transformers may be tied together to achieve a variety of different transformation ratios. An example of three sections connected together is shown in Fig. 6.5. In this circuit the current from the generator splits into four currents going into the transmission lines. Because of the equivalence of the odd-mode currents in each line, these four currents are all equal. The voltages on the load side of each line pair build up from ground to 4 × the input voltage. As a result, for match to occur, $R_L = 16R_G$.

The voltages and currents for a transmission line transformer (TLT) having a wide variety of different interconnections and numbers of transmission lines can

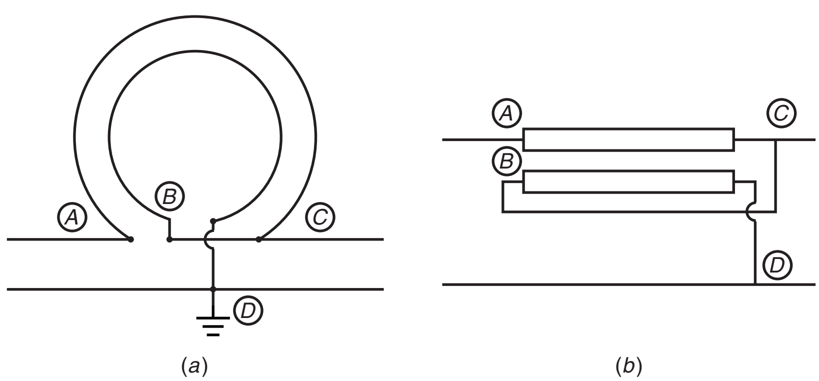

FIGURE 6.3 A physical two-wire transmission line transformer and the equivalent formal representation.

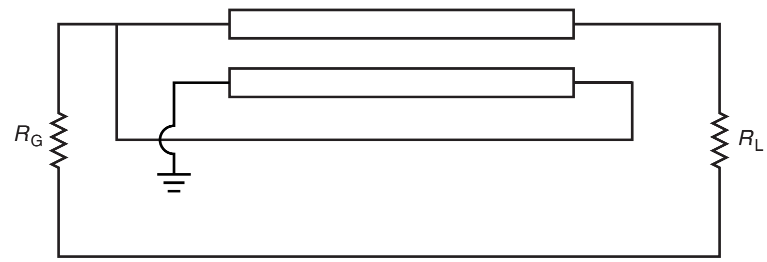

FIGURE 6.4 An alternate transmission line transformer connection.

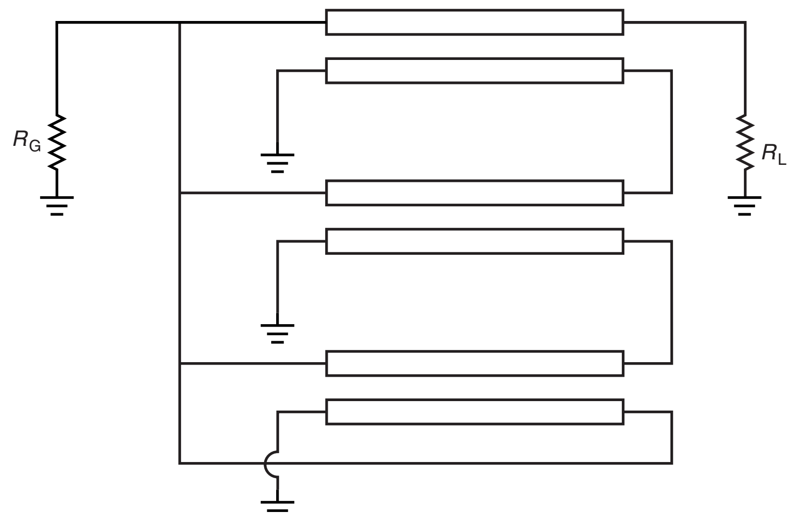

FIGURE 6.5 A 16 : 1 transmission line transformer.

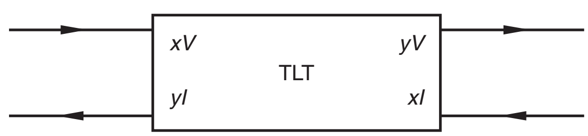

FIGURE 6.6 Symbol for general transmission line transformer.

be represented by the simple diagram in Fig. 6.6 where x and y are integers. The impedance ratios, $R_G = (x/y)^2 R_L$, range from (1 : 1) for a one-transmission line circuit to (1 : 25) for a four-transmission line circuit with a total of 16 different transformation ratios [1]. A variety of transmission line transformer circuits are found in [1] and [2].

6.3 TRANSMISSION LINE TRANSFORMER SYNTHESIS

All the transmission lines in the transmission line transformer shown in Fig. 6.5 have their left-hand sides near the generator connected in parallel and all their right-hand sides near the load connected in series. In this particular circuit there are three transmission lines, and analysis shows that $V_{in} : V_{out} = 1 : 4$, and $R_G : R_L = 1 : 16$. The number of transmission lines, m, is the order of the transformer, so that when all the transmission lines on the generator side are connected in shunt and on the load side in series, the voltage ratio is $V_{in} : V_{out} = 1 : (m + 1)$. Synthesis of impedance transformations of 1 : 4, 1 : 9, 1 : 16, 1 : 25, and so on, are all obvious extensions of the transformer shown in Fig. 6.5. The allowed voltage ratios, which upon being squared, gives the impedance ratios as shown in Table 6.1. To obtain a voltage ratio that is not of the form $1 : (m + 1)$, there is a simple synthesis technique [3]. The voltage ratio is $V_{in} : V_{out} = H : L$, where H is the high value and L the low value. This ratio is decomposed into an $V_{in} : V_{out} = H - L : L$. If now $H - L < L$, this procedure is repeated where $H' = L$ and $L' = H - L$. This ratio is now $V_{out} : V_{in}$, which in turn can be decomposed into $H' - L' : L'$. These steps are repeated until a 1 : 1 ratio is achieved, all along keeping track which ratio that is being done, $V_{in} : V_{out}$ or $V_{out} : V_{in}$.

An example given in [3] illustrates the procedure. If an impedance ratio of $R_G : R_L = 9 : 25$ is desired, the corresponding voltage ratio is $V_{in} : V_{out} = 3 : 5$

Step 1 $H : L = V_{out} : V_{in} = 5 : 3 \rightarrow (5 - 3) : 3 = 2 : 3$

Step 2 $H : L = V_{in} : V_{out} = 3 : 2 \rightarrow (3 - 2) : 2 = 1 : 2$

Step 3 $H : L = V_{out} : V_{in} = 2 : 1 \rightarrow (2 - 1) : 1 = 1 : 1$

Now working backward from step 3, a $V_{in} : V_{out} = 1 : 2$ transmission line transformer is made by connecting two transmission lines in shunt on the input side and series connection on the output side (Fig. 6.7*a*). From step 2, the V_{out} is already 2, so another transmission line is attached to the first pair in shunt on the output side and series on the input side (Fig. 6.7*b*). Finally from step 1, $V_{in} = 3$

TABLE 6.1 Voltage Ratios for Transmission Line Transformers

Number of Lines	1	2	3	4
	1 : 1	2 : 3	3 : 4	4 : 5
	1 : 2	1 : 2	3 : 5	5 : 7
	—	1 : 3	2 : 5	5 : 8
	—	—	1 : 4	4 : 7
	—	—	—	3 : 7
	—	—	—	3 : 8
	—	—	—	2 : 7
	—	—	—	1 : 5

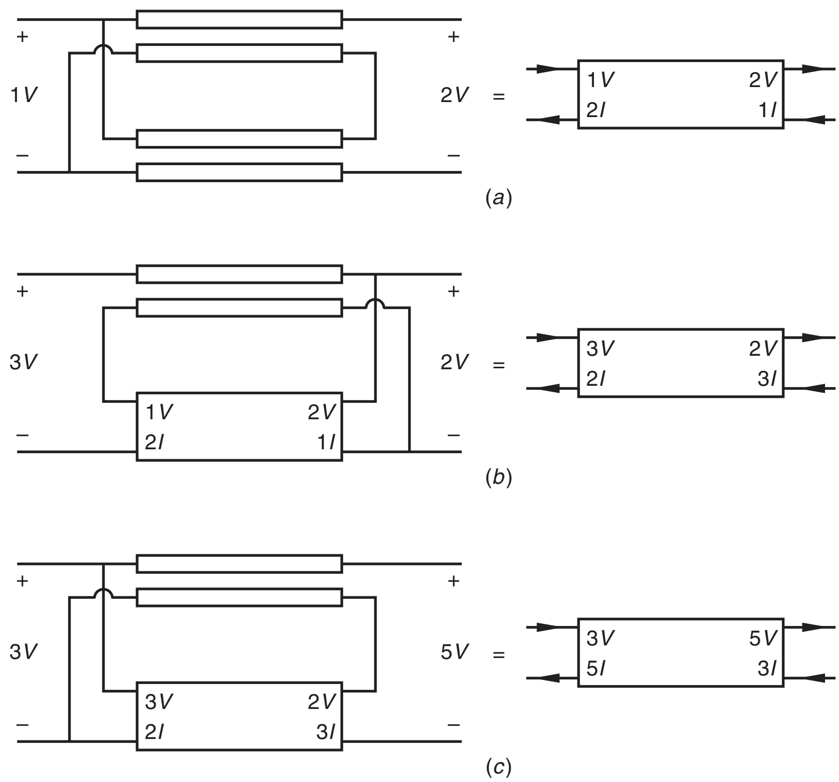

FIGURE 6.7 Step-by-step procedure for synthesis for a desired impedance ratio.

already, so the input is connected in shunt with the another added transmission line and the outputs connected in series (Fig. 6.7*c*). The final design has $V_{in} : V_{out} = 3 : 5$ as desired.

6.4 ELECTRICALLY LONG TRANSMISSION LINE TRANSFORMERS

One of the assumptions given in the previous section was that the electrical length of the transmission lines was short. Because of this the voltages and currents at each end of an individual line could be said to be equal. However, as the the line becomes electrically longer (or the frequency increases), this assumption ceases to be accurate. It is the point of this section to provide a means of determining the amount of error in this assumption. Individual design goals would dictate whether a full frequency domain analysis is needed.

As was pointed out in Chapter 4, the total voltage and current on a transmission line are each expressed as a combination of the forward and backward terms (Fig. 6.8). In this case let V_2 and I_2 represent the voltage and current at the load

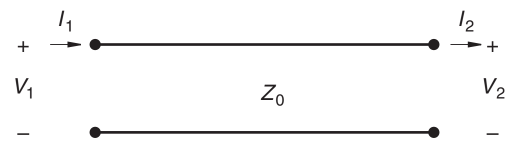

FIGURE 6.8 An electrically long transmission line.

end, where V^+ and V^- are the forward- and backward-traveling voltage waves:

$$V_2 = V^+ + V^- \tag{6.2}$$

$$I_2 = \frac{V^+}{Z_0} - \frac{V^-}{Z_0} \tag{6.3}$$

Assuming that the transmission line is lossless, the voltage and current waves at the input side, 1, are modified by the phase associated with the electrical length of the line:

$$V_1 = V^+ e^{j\theta} + V^- e^{-j\theta} \tag{6.4}$$

$$I_1 = \frac{V^+}{Z_0} e^{j\theta} - \frac{V^-}{Z_0} e^{-j\theta} \tag{6.5}$$

The sign associated with the phase angle, $+\theta$, for V^+ is used because the reference is at port 2 while a positive phase is associated with traveling from left to right. The Euler formula is used in converting the exponentials to sines and cosines. The voltage at the input, V_1, is found in terms of V_2 and I_2 with the help of Eqs. (6.2) and (6.3):

$$V_1 = V_2 \cos\theta + jZ_0 I_2 \sin\theta \tag{6.6}$$

Similarly I_1 can be expressed in terms of the voltage and current at port 2:

$$I_1 = I_2 \cos\theta + j\frac{V_2}{Z_0} \sin\theta \tag{6.7}$$

The 1 : 4 transmission line transformer shown in Fig. 6.4 is now reconsidered in Fig. 6.9 to determine its frequency response. The generator voltage can be expressed in terms of the transmission line voltages and currents:

$$V_g = (I_1 + I_2)R_G + V_1 \tag{6.8}$$

The nontransmission line connections are electrically short. Therefore the output voltage across R_L is $V_o = V_1 + V_2$, and

$$V_g = (I_1 + I_2)R_G + I_2 R_L - V_2 \tag{6.9}$$

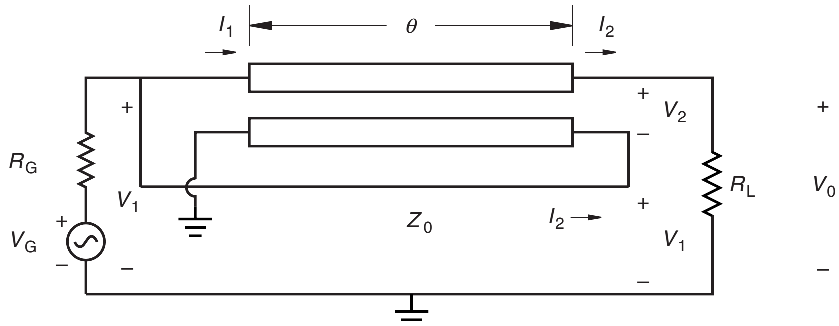

FIGURE 6.9 An electrically long 1 : 4 transmission line transformer.

In Eqs. (6.9), (6.6), and (6.7), V_1 is replaced by $I_2R_L - V_2$ to give three equations in the three unknowns I_1, I_2, and V_2:

$$V_G = I_1R_G + I_2(R_G + R_L) - V_2 \tag{6.10}$$

$$0 = 0 + I_2(jZ_0 \sin\theta - R_L) + V_2(1 + \cos\theta) \tag{6.11}$$

$$0 = -I_1 + I_2 \cos\theta + j\frac{V_2}{Z_0}\sin\theta \tag{6.12}$$

The determinate of these set of equations is

$$\Delta = -2R_G(1 + \cos\theta) - R_L \cos\theta + j \sin\theta \left(\frac{-R_GR_L}{Z_0} - Z_0\right) \tag{6.13}$$

and the current I_2 is

$$I_2 = \frac{-V_G(1 + \cos\theta)}{\Delta} \tag{6.14}$$

Consequently the power delivered to the load from the source voltage is

$$P_o = \frac{1}{2}|I_2|^2R_L = \frac{1}{2}\frac{|V_g|^2(1 + \cos\theta)^2R_L}{[2R_G(1 + \cos\theta) + R_L \cos\theta]^2 + [(R_GR_L + Z_0^2/Z_0^2])\sin^2\theta} \tag{6.15}$$

Now the particular value of R_L that guarantees maximum power transfer into the load is found by maximizing Eq. (6.15). Let D represent the denominator in Eq. (6.15):

$$\frac{dP_o}{dR_L} = 0 = \frac{1}{2}|V_G|^2\frac{(1 + \cos\theta)^2}{D} \times \left\{1 - \frac{R_L}{D}\left[2[2R_G(1 + \cos\theta) + R_L \cos\theta]\cos\theta + [\cdots]\sin^2\theta\ \right]\right\} \tag{6.16}$$

In the low-frequency limit where $\theta \to 0$, $R_L = 4R_G$. The optimum characteristic impedance is found by maximization P_o with respect to Z_0, while this time keeping the line length $\neq 0$. The result is not surprising, as it is the geometric mean between the generator and load resistance:

$$Z_0 = 2R_G \tag{6.17}$$

The output power then when $Z_0 = 2R_G$ and $R_L = 4R_G$ is

$$P_o = \frac{1}{2}\frac{|V_G|^2(1+\cos\theta)^2}{R_G(1+3\cos\theta)^2 + 4R_G\sin^2\theta} \tag{6.18}$$

This reduces to the usual form for the available power when $\theta \to 0$.

More complicated transmission line transformers might benefit from using SPICE to analyze the circuit. The analysis above gives a clue to how the values of Z_0 and the relative values of R_G and R_L might be chosen with the help of a low frequency analysis.

As an example consider the circuit in Fig. 6.9 again where $R_G = 50\ \Omega$ so that $R_L = 200\ \Omega$ and $Z_0 = \sqrt{50 \cdot 200} = 100\ \Omega$, and the electrical length of the transformer is 0.4 wavelength long at 1.5 GHz. The plot in Fig. 6.10 is the return loss (= 20 log of the reflection coefficient) as seen by the generator voltage V_G.

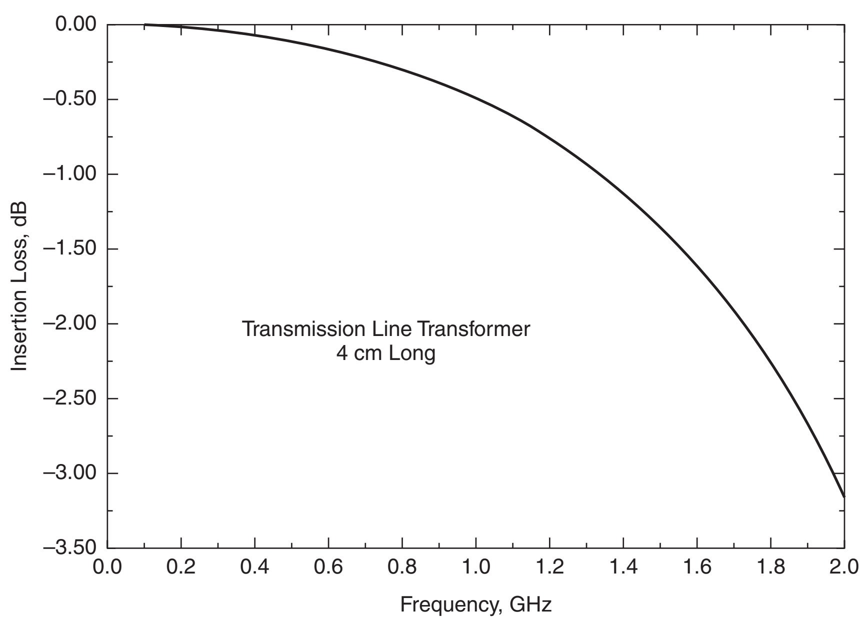

FIGURE 6.10 Return loss for the frequency dependent transmission line transformer of Fig. 6.9.

The SPICE net list used to analyze this circuit makes use of the conversion of voltages to S parameters:

```
Analysis of a circuit for S11 and S21
*
* R01 and R02 are input and output resistance levels.
* RL is the load resistance. The load may be
  supplemented
* with additional elements.
*.PARAM R01=50, R02=50. RLOAD=50. IN1=-1/R01
.PARAM R01=50, R02=200. RLOAD=200. IIN=-1/R01
.FUNC N(R01,R02) SQRT(R02/R01)
R01   1    0   R01
VIN   10   11  AC   1
GI1   1    0   VALUE=-V(10,11)/R01
*GI1  1    0   10  11  "-1/R01"
E11   10   0   1   0   2
R11   11   0   1
Xcircuit   1   2   TLTCKT
RL    2    0   RLOAD
E21   21   0   VALUE=V(2)*2/N(R01,R02)
*  n = SQRT(R02/R01)
*E21   21   0   2   0   "2/n"
R21   21   0   1
*
.SUBCKT TLTCKT   1  4
* Input side
* 4 cm = .1333 wavelength at 1 GHz
TLT4  1    0   4   1   Z0=200  F=1GHz  NL=.1333
* Output side
.ENDS   TLTCKT
* Code for S11 and S21
*.AC DEC "num"  "f1"   "f2"
.AC  LIN  301  .1MEG   2GHZ
.PROBE V(11)  V(21)
.END
```

6.5 BALUNS

A balun (balanced–unbalanced) is a circuit that transforms a balanced transmission line to an unbalanced transmission. An example of a balanced line is the two-wire transmission line. An unbalanced line is one where one of the lines is grounded, such as in coaxial line or microstrip. One situation where this is important is in feeding a dipole antenna with a coaxial line where the antenna is balanced and the coaxial line is unbalanced. One simple structure is shown in Fig. 6.11 where the difference between the inputs of the antenna is forced to be 180° by addition of a half wavelength line between them. At RF frequencies, a more practical way to perform this same function is to use a transmission line transformer as shown in the example of the 1 : 1 balun in Fig. 6.12*a*. There is no specified ground on the right-hand side of this circuit, but since the voltage difference on the input side is V, the voltage across the load must also be V.

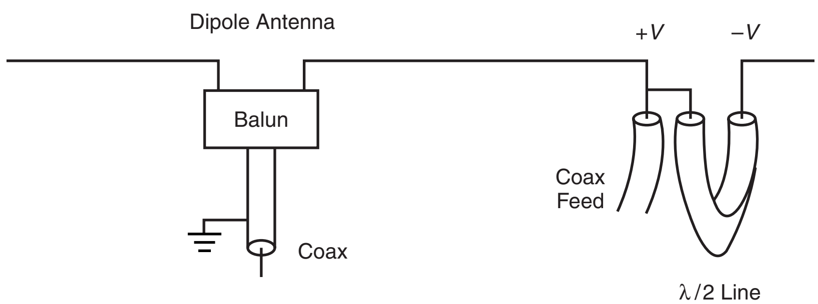

FIGURE 6.11 Balun example used for dipole antenna.

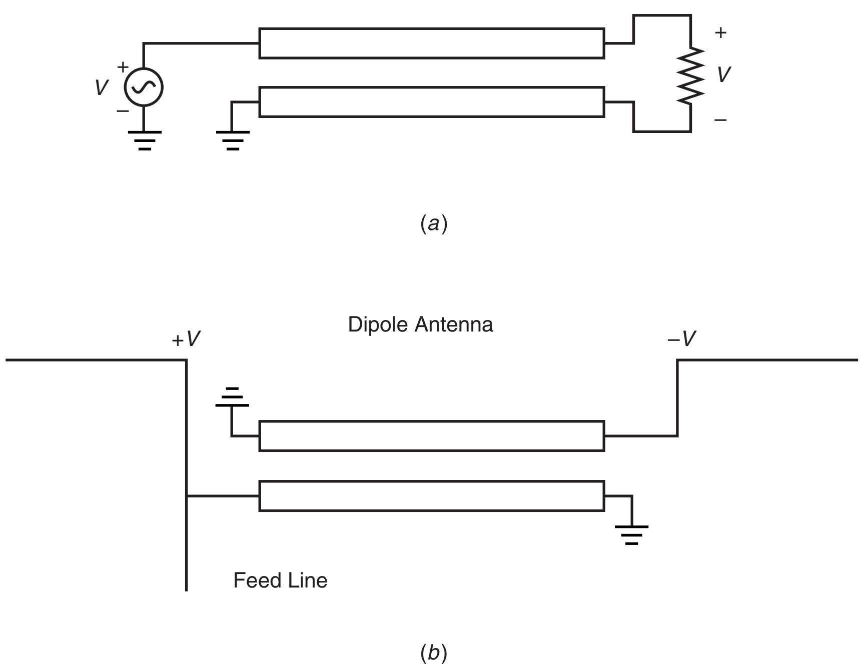

FIGURE 6.12 (*a*) Transmission line transformer implementation of a (1 : 1) balun, and (*b*) grounding one side gives a $+V$ and $-V$ to the two sides of the dipole antenna.

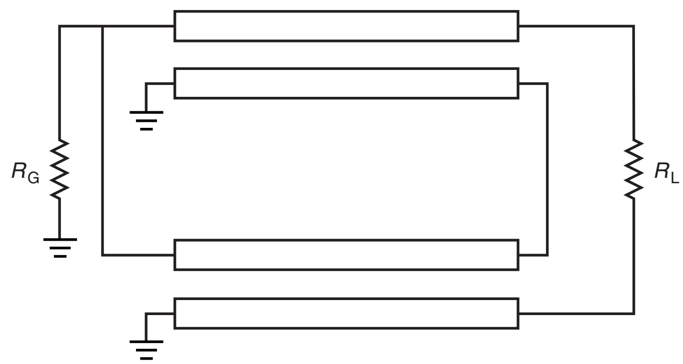

FIGURE 6.13 A balun with a $R_G : R_L = 1 : 4$ impedance ratio.

For the dipole application, where a $+V$ is needed on one side and $-V$ on the other side, one of the output sides can be grounded as indicated in Fig. 6.12*b*. The ($R_G : R_L = 1 : 4$) balun in Fig. 6.13 shows that impedance matching and changing to a balanced line can be accomplished with a balun. Analysis of this circuit may be aided by assuming some voltage, V_x, at the low side of R_L. When the voltage at the upper side of R_L is found, it also contains V_x. The difference between the lower and upper sides of R_L removes the V_x.

6.6 DIVIDERS AND COMBINERS

Transmission lines can be used to design power dividers and power combiners. These are particularly important in design of high-power solid state RF amplifiers where the input can be split between several amplifiers or where the outputs of several amplifiers may be effectively combined into one load. A very simple two-way power divider is shown in Fig. 6.14. In this circuit $R_L = 2R_G$, and the

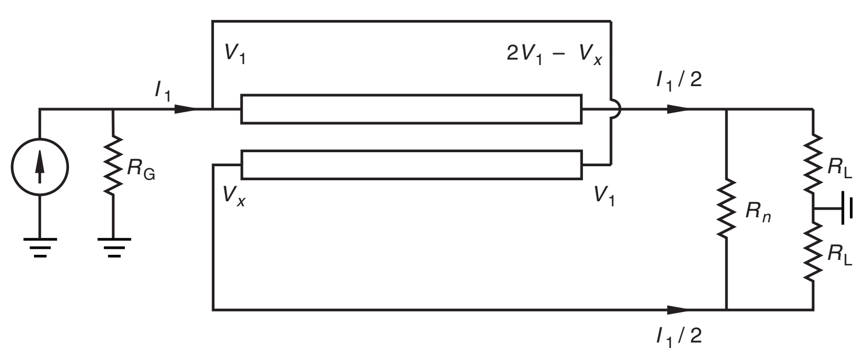

FIGURE 6.14 A two-way power divider.

transmission line characteristic impedance is designed to be $Z_0 = \sqrt{2}R_G$. The current in R_n ordinarily would be 0 because of equal voltages on either side of that resistance. Under unbalanced load conditions, R_n can absorb some of the unbalanced power and thus protect whatever the load is. When the two loads are both $2R_G$. The input voltage is V_1 on the top conductor, and the voltage on the lower conductor is V_x on the left side. On the right-hand side the lower conductor is V_1, and so the top conductor must be $2V_1 - V_x$ to ensure that both sides of the transmission line have the same voltage across the terminals, that is, $V_1 - V_x$. Since the current flowing through the upper load resistor and the lower load resistor must be the same, the voltage on either side of R_n is the same. Consequently $2V_1 - V_x = V_x$ or $V_x = V_1$, so the voltage to current ratio at the load is

$$R_L = \frac{V_1}{I_1/2} = 2R_G \tag{6.19}$$

A two-way 180° power combiner shown in Fig. 6.15 makes use of a hybrid coupler and a balun. The resistor R_n is used to dissipate power when the two inputs are not exactly equal amplitude or exactly 180° out of phase so that matched loading for the two sources is maintained. For example, consider when $I_1 = I_2$, as shown in Fig. 6.15, so that I_1 is entering the hybrid and I_2 is leaving the hybrid. The current flowing through the load, R_L, is I_0. The current flowing into the hybrid transmission line from the top is $I_1 - I_0$, while the current

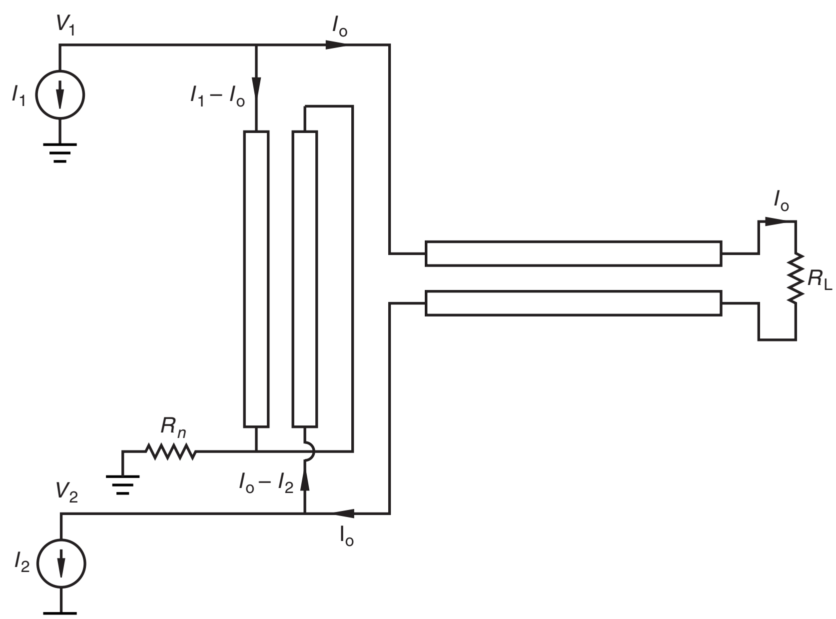

FIGURE 6.15 A two-way 180° power combiner.

flowing from the bottom is $I_0 - I_2$. The odd-mode current in the transmission line forces is

$$I_1 - I_0 = I_0 - I_2$$

or

$$I_0 = I_1 \tag{6.20}$$

All the current goes through the balun, and no current flows through the hybrid. The current through R_n is therefore 0 leading to $V_x = 0$. The voltage difference between the two ends of the transmission lines of the hybrid is the same, which implies that

$$V_1 - V_x = V_x - V_2$$

or

$$V_1 = -V_2 \tag{6.21}$$

and

$$V_0 = V_1 - V_2 = 2V_1 \tag{6.22}$$

The matching load resistance is then

$$\frac{V_0}{I_0} = R_\text{L} = 2R_\text{G} \tag{6.23}$$

When I_1 and I_2 are both entering the circuit so that $I_1 = -I_2$, and $V_1 = V_2$, then voltages across the top and bottom of the transmission line in the hybrid circuit of Fig. 6.15 are

$$V_1 - V_x = V_x - V_2$$

or

$$V_x = V_1 \tag{6.24}$$

The voltage across the load is $V_0 = 0$ and $I_0 = 0$. The current in the hybrid transmission line is I_1, so the current flowing through R_n is $2I_1$:

$$R_n = \frac{V_x}{2I_1} = \frac{V_1}{2I_1} = \frac{1}{2}R_\text{G} \tag{6.25}$$

The choices for R_L and R_n ensure impedance matching for an arbitrary phase relationship between I_1 and I_2. Optimum performance would be expected if the characteristic impedances of the transmission lines were

$$Z_{0\text{-balun}} = \sqrt{2}R_\text{G} \tag{6.26}$$

$$Z_{0\text{-hybrid}} = \frac{R_\text{G}}{\sqrt{2}} \tag{6.27}$$